Abstract: This review is concerned with nanoscale effects in highly transparent dielectric photonic structures fabricated from optical fibers. In contrast to those in plasmonics, these structures do not contain metal particles, wires, or films with nanoscale dimensions. Nevertheless, a nanoscale perturbation of the fiber radius can significantly alter their performance. This paper consists of three parts. The first part considers propagation of light in thin optical fibers (microfibers) having the radius of the order of 100 nanometers to 1 micron. The fundamental mode propagating along a microfiber has an evanescent field which may be strongly expanded into the external area. Then, the cross-sectional dimensions of the mode and transmission losses are very sensitive to small variations of the microfiber radius. Under certain conditions, a change of just a few nanometers in the microfiber radius can significantly affect its transmission characteristics and, in particular, lead to the transition from the waveguiding to non-waveguiding regime. The second part of the review considers slow propagation of whispering gallery modes in fibers having the radius of the order of 10 – 100 microns. The propagation of these modes along the fiber axis is so slow that they can be governed by extremely small nanoscale changes of the optical fiber radius. This phenomenon is exploited in SNAP (surface nanoscale axial photonics), a new platform for fabrication of miniature super-low-loss photonic integrated circuits with unprecedented sub-angstrom precision. The SNAP theory and applications are overviewed. The third part of this review describes methods of characterization of the radius variation of microfibers and regular optical fibers with sub-nanometer precision.

Keywords: microfiber; microresonator; nanophotonics.
2 Optical microfibers with the radius of the order of 100 nm–1 μm

In this section, the transmission properties of very thin optical fibers with a radius of the order of 100 nanometers to 1 micron are considered. These fibers are referred to as microfibers. Optical microfibers are proposed as building blocks for micro-optical devices having applications in photonics, physics, chemistry, and biology [4–23]. Important advantages of microfibers are the potential for compact assembly in three dimensions, possibility of strong coupling to the environment and/or localization of radiation, and low transmission loss.

As shown below, the transmission spectrum of a very thin microfiber having a radius of the order of 100 nm is extremely sensitive to nanoscale variation of its radius. Subsection 2.1 briefly considers uniform microfibers. In Subsection 2.2, the theory of tunneling from microfiber tapers is outlined. As shown in Subsection 2.3, this theory predicts the fundamental limit for microfiber radii below which they cannot waveguide light at all. The threshold between waveguiding and non-waveguiding microfiber radii is only a few nanometers. The experimental confirmation of the theory of tunneling for microfiber tapers and of the fundamental limit for waveguidance is given in Subsection 2.4.

2.1 Evanescent field of a uniform microfiber

If the optical microfiber diameter is close to or greater than the wavelength of light λ, then the fundamental mode is primarily localized inside the microfiber as shown in Figure 1(A). However, for smaller diameters, considerable fraction of light propagates outside the microfiber, as illustrated in Figure 1(B). The characteristic width of the mode W is determined by the transverse propagation constant γ₀ as W = 2γ₀/γ. For example, silica microfibers with small radius r₀ << λ and refractive index n = 1.46 [24, 25]:

\[
\gamma_0 = \frac{2}{W} = \frac{1.65}{r_0} \exp \left( -0.0713 \frac{\lambda^2}{r_0^2} \right).
\]  

(1)

From this equation, the mode width increases dramatically with the decreasing of the microfiber radius. The dependence of the mode width on the microfiber diameter found from this equation is shown in Figure 1(C). From this figure, for a microfiber with radius r₀ = 100 nm, the fundamental mode width achieves the intuitively unrealistic value of 1 m. Can a microfiber of 100 nm radius support light with the characteristic transverse dimension of a meter? Formal theory of uniform waveguides cannot answer this question, which requires the consideration following below.

2.2 Optics of tunneling from an adiabatically tapered microfiber

Theoretically, the fundamental mode of a uniform dielectric waveguide considered in the previous subsection can be supported by a microfiber independently of its thickness. However, the waveguiding ability of a very low loss silica microfiber is primarily constrained by geometric nonuniformities. Following [24–27] the term “waveguide” is applied here only to such a waveguide that exhibits
relatively small losses. In practice, the transmission loss of a waveguide is determined by its own optical properties as well as by the way the input and output of light is performed. In most cases, there exists a way to minimize the input and output losses down to relatively small values. For this reason, these losses can usually be separated from the losses of the waveguide itself. However, for a very thin waveguide and, in particular, for an optical microfiber, the transmission loss is primarily determined by input and output losses, which cannot be reduced significantly [24].

The hypothesis proposed in [24] was that the smallest input/output losses are achieved for a thin microfiber with radius \( r_0 \) [Figure 1(B)] if the input/output microfiber [Figure 1(A)] is connected to this microfiber adiabatically. Then, to determine the smallest transmission losses of a thin microfiber, it is necessary to determine losses of an adiabatic taper. As compared to the well-known problem of transmission losses in a bent uniform microfiber which allows approximate separation of variables and is reduced to tunneling through a potential barrier [28], the problem of losses in adiabatically tapered microfibers is more complicated. The optics of light propagation along an adiabatic taper is visualized in Figure 2. Usually, it is determined by the splitting of the fundamental mode near the focal circumferences shown in this figure. In the neighborhood of each focal circumference, the input mode is split into two components. The component which is closer to the microfiber axis continues as the fundamental mode, while another component contributes to the exponentially small radiation loss. The focal circumferences are responsible for splitting off the radiating components. The interference of these components with each other and the guiding mode gives rise to a complex behavior of the evanescent field. As an example, let us consider a taper which is determined by the following variation of the transverse propagation constant along the microfiber axis \( z \) [Figure 3(A)]:

\[
\gamma(z) = \gamma_0 + \frac{\gamma_1 \gamma_0}{1 + \exp(-z/L)}.
\]

Here \( \gamma_0 \) and \( \gamma_1 \) are the transverse propagation constants to the far left and far right hand side of this taper, respectively, and \( L \) is the characteristic length of the tapered region. Figure 3(B) shows the surface plot of electromagnetic field intensity which is calculated using the beam propagating method for the taper determined by Eq. (2) with parameters determined in the figure caption [26]. The upper arrow shows the direction of light propagation. The quasiperiodic dips with vanishing field intensity are explained as follows. In the neighborhood of the focal point indicated by a small circle, the radiating component of the field splits off and localizes along a curve starting near the focal point, which is slowly separating from the microfiber axis and going to infinity. From this line down, the radiating component exponentially decreases, while the guiding component exponentially increases. Therefore, at a certain distance from the microfiber axis, these components become equal in magnitude. Since the propagation constants of these components are different, their interference causes strong oscillations of the field.

**Figure 2** Characteristic behavior of the fundamental mode propagating along the tapered microfiber. After passing the focal points indicated by black dots (which are the projections of focal circumferences in 3D space), the mode splits into guiding and radiating components.

**Figure 3** Distribution of the electromagnetic field intensity near the microfiber taper defined by Eq. (2) with parameters \( \gamma_0 = 0.2 \mu m^{-1} \), \( \gamma_1 = 0.4 \mu m^{-1} \), and \( L = 50 \mu m \), for radiation wavelength \( \lambda = 1.5 \mu m \) [26]. Calculations are performed using the beam propagation method.
2.3 How thin can a microfiber be and still guide light?

The theory developed in [24–26] and outlined in the previous subsection allows one to determine the propagation loss of adiabatically tapered microfibers in a relatively simple analytical form. In particular, in the case of a strongly tapered microfiber taper defined by Eq. (2) with \( \gamma_i > \gamma_0 \), the propagation loss is independent of the transversal propagation constant \( \gamma_i \) of the thicker part of the taper [26]:

\[
P = \frac{\pi^{1/2}}{4S^{1/2}} \exp(-S), \quad S = \frac{1}{2}L\gamma_i^2.
\]

(3)

According to the definition of a waveguide given in Subsection 1.2, a thin microfiber is waveguiding only if \( P < 1 \), while the condition \( P = 1 \) corresponds to the threshold between the waveguiding and non-waveguiding behavior. In Figure 4, the dependencies of the propagation loss on the microfiber radius \( r_0 \) are presented for different characteristic taper lengths: small \( L = 100 \mu m \), intermediate \( L = 10 \) cm, and dramatically large \( L = 100 \) km. Calculations are performed using Eqs. (1)–(3). This figure demonstrates that, indeed, there is a dramatic threshold behavior of transmission loss (curves 1–3). Mathematically, the threshold is caused by the superfast double-exponential behavior of the transmission power in the microfiber radius (see Eqs. (1) and (3)). Crucially, while the threshold values of the microfiber radius for a relatively short \( L = 100 \mu m \) is 200 nm, it is only two times less than the threshold radius 100 nm for the gigantic \( L = 100 \) km. Figure 4 demonstrates the fundamental limit for the radius of the waveguiding microfiber at wavelength \( \lambda = 1.5 \mu m \) equal to \( r_{\text{min}} = 100 \) nm. For the arbitrary wavelength \( \lambda \) and refractive index \( n \) of the microfiber, the smallest possible radius of the waveguiding microfiber is found as

\[
r_{\text{min}} = 0.1 \lambda / n.
\]

(4)

From Figure 4, the threshold corresponds to a change in the microfiber radius of only several nanometers, which indicates a significant variation of the transmission spectrum with nanoscale change of the microfiber radius. These predictions have found excellent experimental confirmation both for the silica microfibers considered in the next subsection and for THz fibers [29].

2.4 Transmission threshold of the optical microfiber waveguide: experiment

In this subsection, following Ref. [27], it is shown that the experimentally determined radius of the thinnest possible waveguiding microfiber is in good agreement with predictions of the theory of adiabatic microfiber tapers described in Subsection 2.3. In the experiment considered in [27], the microfiber represents a waist of a biconical fiber taper. It was fabricated using the indirect laser heating method [8]. A conventional 62.5 \( \mu m \) radius optical fiber was placed into a sapphire tube serving as a microfurnace which was heated by an external CO₂ laser beam and drawn down to the submicron cross-section dimensions. The drawing method consists of periodical translations of the fiber into opposite directions with respect to the laser beam and simultaneous pulling in opposite directions [9]. After a cycle of drawing, the fiber radius is set to be reduced by \( f = 0.75 \) so that after the \( N \)th cycle the microfiber radius was:

\[
r^{(N)} = R^{(0)} \times 0.75^N \mu m.
\]

(5)

The transmission loss of the biconical taper was calculated with the theory described in Subsection 2.3. The tapered sections of fabricated microfiber were approximated by Eqs. (1) and (2) with the characteristic length parameter \( L = 250 \mu m \). This parameter was determined by fitting the experimentally measured variation of the microfiber diameter. The method of measurement, which allows the detection of the radius variation with angstrom-scale precision, is described in Subsection 4.1. In Figure 5(A), the transmission loss defined using Eqs. (1) and (3) is plotted as a function of the microfiber radius for different wavelengths. This plot determines the microfiber radii, in the neighborhood of which the transmission loss starts a noticeable deviation from zero. The solid and dashed lines correspond to the sizes of the tapered region \( L = 250 \mu m \) and \( L = 500 \mu m \), respectively. The table below Figure 5(A) determines the closest microfiber radii corresponding to the integer \( N \) in Eq. (5). The predictions of Figure 5(A) were verified experimentally by monitoring the transmission power of the microfiber taper in the process of drawing.
Figure 5(B) shows the results of measurements [27]. The horizontal axis in this figure is the effective microfiber radius calculated by the continuation of Eq. (5) to the non-integer \( N \). The characteristic wavy behavior of the spectra near the threshold radii is explained by temporal nonuniformity of the drawing process. Comparison of the theoretical calculation in Figure 5(A) with the experimental data in Figure 5(B) demonstrates a remarkable agreement of the values for the threshold radii.

3 Optical fibers with the radius of the order of 10–100 \( \mu \)m

In conventional silica optical fibers, light propagates along the interior fiber core and has the propagation constant \( \beta_0 = \lambda / (2 \pi n_f) \) and speed \( v_0 = c / n_f \), where \( c \) is the speed of light, \( \lambda \) is the radiation wavelength, and \( n_f \) is the fiber refractive index. In contrast, the whispering gallery modes (WGMs) considered in this section circulate along the fiber surface and slowly propagate along the fiber axis with propagation constant \( \beta \), which is much smaller than the propagation constant along the fiber core, \( \beta < \beta_0 \). The speed of axial propagation of these modes \( v \) is much smaller than the speed of light in the fiber material \( v_0 \). The central idea of SNAP (Surface Nanoscale Axial Photonics) [30–37] reviewed in this section is to exploit the sensitivity of WGMs to extremely small variations of the effective fiber radius, which is proportional to the variation of the physical fiber radius \( \Delta r(z) = r(z) - r_0 \) and refractive index \( \Delta n_f(z) = n_f(z) - n_f_0 \):

\[
\Delta r_{\text{eff}}(z) = r_0 \left( \frac{\Delta n_f(z)}{n_f_0} + \frac{\Delta r(z)}{r_0} \right),
\]

It is assumed that the radius and index variations in Eq. (6) are appropriately averaged over the fiber cross-section. Figure 6 gives a simple explanation of why an extremely small local variation of the optical fiber radius can completely localize a WGM. In a conventional optical microresonator having the shape of an ellipsoid with radii \( r \) and \( R \), the width \( w \) of the fundamental WGM illustrated in Figure 6(A) is

\[
w = \left( \frac{\lambda}{2 \pi n_f} (rR) \right)^{1/4},
\]

(see, e.g., [38]). For example, for a silica microsphere with refractive index \( n_f = 1.46 \) and radius \( r = R = 20 \) \( \mu \)m, at wavelength \( \lambda = 1.5 \) \( \mu \)m, we have \( w = 0.7 \) \( \mu \)m. At the same wavelength, for a silica fiber with radius \( r = 20 \) \( \mu \)m and...
an extremely small axial deformation corresponding to a gigantic radius \( R=1 \, \text{m} \) illustrated in Figure 6(B), the width of the fundamental WGM found from Eq. (7) is still remarkably small, only \( w=11 \, \text{μm} \). In the region of localization, variation of the local fiber radius is only \( w^2/R=1.2 \, \text{Å} \). Thus, an angstrom-scale local variation of the fiber radius can completely localize light along a microscale length of the optical fiber. This effect enables the fabrication of chains of coupled microresonators at the surface of an optical fiber illustrated in Figure 6(C) as well as other SNAP devices.

Crucially, since SNAP devices are fabricated at the very smooth surface of an extremely low loss silica fiber, the attenuation of light in SNAP devices can be more than two orders of magnitude smaller than in miniature photonics circuits with similar dimensions. In contrast to resonant planar photonic circuits and photonic crystal circuits, coupling between elements of these devices is interfaceless and does not introduce additional losses. Furthermore, the fabrication precision demonstrated for SNAP devices is two orders of magnitude higher than that achieved in the state of the art photonics fabrication technologies [39–44].

The general description of the behavior of WGMs propagating along the optical fiber with nanoscale radius variation is given in Subsection 3.1. Subsections 3.2–3.4 review methods of fabrication of SNAP devices with sub-angstrom precision, and Subsection 3.5 compares the SNAP platform with state of the art photonics technologies developed to date.

### 3.1 “Quantum mechanics” of light at the surface of an optical fiber

We consider slow propagation of WGMs along the surface of an optical fiber with the propagation constant \( \beta(\lambda, z) \ll \beta_0 \), depending on the radiation wavelength \( \lambda \) and axial coordinate \( z \). Variation of the SNAP fiber radius and refractive index along the fiber axis \( z \) is so small and slow that the propagation of light is purely adiabatic. In the cylindrical frame of reference \((\rho, \phi, z)\), a WGM can be found as \( \exp(i m \phi) \overline{\xi}_q(\rho) \overline{\Psi}(z) \) [28] where \( m \) and \( q \) are the orbital and radial quantum numbers, and function \( \overline{\Psi}(z) \) satisfies the Schrödinger equation [30, 31]:

\[
\frac{d^2 \overline{\Psi}}{dz^2} + (E(\lambda) - V(z)) \overline{\Psi} = 0,
\]

where energy is proportional to wavelength variation, and potential is proportional to the effective radius variation:

\[
E(\lambda) = -2\beta_0^2 \frac{\lambda - \lambda_{\text{res}}}{\lambda_{\text{res}}} - i \gamma_{\text{res}} \overline{\Psi}(z), \quad V(z) = -2\beta_0^2 \frac{\Delta r_{\text{eff}}(z)}{r_0}.
\]

In this equation, \( \lambda_{\text{res}} \) is the resonant wavelength, and \( \gamma_{\text{res}} \) determines the material attenuation. For silica fibers, parameter \( \gamma_{\text{res}} \) can be as small as \( 10^{-3} \, \text{pm} \). Figure 7 illustrates the relation between fiber radius variation and potential in the Schrödinger equation, Eq. (8). Usually, WGMs in an optical fiber are excited by the evanescent field of a transverse microfiber as shown in Figure 7(A). A WGM with wavelength \( \lambda \) has the propagation constant defined from Eq. (8):

\[
\beta(\lambda, z) = \sqrt{E(\lambda) - V(z)}.
\]

Formally, the propagation constant can be very small (only limited by the value of material losses \( \gamma_{\text{res}} \)). In contrast to the slow light photonic devices, which are conventionally engineered from periodic structures (coupled microresonators [39–44] and photonic crystals [41, 45]), the SNAP fiber exhibits a naturally slow light created in the absence of an optical medium with a periodically modulated refractive index. The simplest derivation of Eqs. (8)–(10) assumes the original cylindrical symmetry of the fiber. However, these equations are also valid for realistic fibers with slightly asymmetric cross-sections. In this case, the local effective fiber radius is calculated by averaging over azimuthal angle.

Figure 7(B) illustrates the magnified fiber radius variation of a SNAP fiber, and Figure 7(C) shows the dependence of the potential in the Schrödinger equation, Eq. (8), proportional to this variation. The position of energy \( E \) (proportional to the wavelength variation) in Figure 7(C) exhibits basic building blocks of SNAP corresponding to phenomena well known in elementary quantum mechanics [46]. Region 1 is a bottle resonator [47] in Figure 7(B), which corresponds to a quantum well in Figure 7(C). If the input/output microfiber is situated in this region, it excites the WGMs, which are localized between turning points \( z_{i1} \) and \( z_{i2} \) at a discrete series of wavelengths. Region 2 in Figure 7(B) is a very shallow concave fiber waist corresponding to a potential barrier in Figure 7(C). In this region, under the barrier, for \( E<V(z) \), the WGM experiences exponential decay. Alternatively, above the barrier, for \( E>V(z) \), the WGM is delocalized. Finally, if the input/output microfiber is situated in region 3 has a monotonically increasing fiber radius, then a WGM propagating towards the barrier will reflect from the turning point \( z_{i1} \) and interfere with itself. It can be shown that, in this case, at discrete wavelengths, when the condition of destructive interference
is fulfilled, the distribution of light is fully localized between the turning point $z_t$ and microfiber position $z_1$. Remarkably, in this case, a single point of contact with a microfiber completely halts light propagating along the surface of an optical fiber \[30, 31\].

The transmission amplitude of a SNAP fiber coupled to a single input/output microfiber illustrated in Figure 7(A) is \[35\]:

\[
S(\lambda, z) = S_0 \cdot \frac{|C| G_\lambda(\lambda, z_1, z_i)}{1 + DG_\lambda(\lambda, z_1, z_i)},
\]

where $S_0$ is the out-of-resonance amplitude, $C$ and $D$ are the SNAP fiber/microfiber coupling parameters, and $G(\lambda, z_1, z_i)$ is the Green’s function of the Schrödinger equation, Eq. (8).

Eqs. (8)–(11) fully describe slow propagation of WGMs along the surface of an optical fiber and can be used to create a wide range of SNAP devices. The SNAP fiber can also be coupled to more than one transverse input/output microfiber. In such integrated assemblies, the field distribution along the SNAP device as well as its transmission spectra are described by the general theory developed in \[35\]. This review is confined to the consideration of SNAP devices coupled to a single input/output microfiber.

Importantly, for slow propagation of WGMs considered here, the characteristic axial wavelength is much larger than the wavelength of light and, typically, has the order of a few tens of microns. This dramatically simplifies fabrication of SNAP devices considered in the next subsection and, in particular, the microresonators considered in Subsections 3.3 and 3.4. In addition, this makes the experimental characterization of SNAP devices described in Subsection 4.2 much easier.

### 3.2 Methods of fabrication of SNAP devices

A challenging problem of accurate and reproducible modification of the optical fiber effective radius at nanoscale precision is critical for the establishing of SNAP as a practical technology. This problem was solved in \[32\] using the IR (CO$_2$ laser) and UV (248 nm excimer laser) beam exposures. It has been shown that it is possible to fabricate coupled SNAP microresonators \[32–36\] and other devices \[37\] at the surface of an optical fiber with sub-angstrom precision.

![Figure 8](image_url)
A general method of precise variation of the effective radius of an optical fiber is based on local annealing of the fiber surface performed with a CO\textsubscript{2} laser beam [Figure 8(A)]. It is known that an optical fiber is drawn under certain tension which is frozen in after it cools down [48–50]. It was found that the release of this tension by short-time heating using, e.g., a CO\textsubscript{2} laser beam, causes a nanoscale variation in the fiber effective radius. Depending on the power and duration of the exposure (typically of the order of 1 s), it is possible to reproducibly introduce these variations with sub-angstrom precision (see Subsections 3.3 and 3.4).

Another method for the fabrication of SNAP devices exploits photosensitive fibers. It is known that UV radiation affects both the refractive index and the density of the fiber (see, e.g., [51]). This effect can be used for local variation of the effective fiber radius. In particular, a series of coupled SNAP microresonators can be fabricated along the Ge-doped fiber by exposure to UV radiation through an amplitude mask as illustrated in Figure 8(B). This approach allows the fabrication of very long chains of coupled microresonators with sub-angstrom precision [32, 34]. A combination of methods for the local modification of photosensitive optical fiber radii exploiting both UV and CO\textsubscript{2} exposures has been also demonstrated [34].

Both methods illustrated in Figure 8 allow for the fabrication of super-low-loss complex photonic circuits at the optical fiber surface with sub-angstrom precision. In the experiments below, we consider SNAP devices which consist of a series of coupled microresonators (Subsection 3.3) and of a bottle resonator with a parabolic radius variation (Subsection 3.4).

### 3.3 Fabrication of long coupled microresonator chains with sub-angstrom precision

As described in the previous subsection, the nanoscale modification of the effective radius of an optical fiber can be achieved with a focused CO\textsubscript{2} laser beam. The beam can be translated along the fiber with a varying velocity to introduce the predetermined radius variation. The simplest approach exploits application of multiple discrete laser shots, which can be used both for the introduction of the required variation and its correction. This approach was developed in [36] for the fabrication of coupled microresonator chains.

The experimental setup of Ref. [36] consisted of the SNAP fiber exposure section [Figure 8(A)] and the fiber characterization section described below in Subsection 4.2. The input beam power and the exposure time was controlled by a laser power controller and a shutter with a 1 ms switching time. After the exposure, the SNAP fiber was characterized at the measurement section with the microlens fiber scanning method [Figure 13(A)] [52, 53]. The spacing between the measurement points was set to 10 μm. The 30 radial bumps shown in Figure 9(A) that were originally

![Figure 9](image-url) (A) The surface plot of the transmission spectrum of the originally-fabricated 30 microresonator chain. (B) The transmission spectrum plot of this chain after calibration exposure. (C) The transmission spectrum plot of the corrected chain. (D) The transmission spectrum plot obtained by theoretical modeling of the fabricated microresonator chain. The white solid curve is the calculated effective radius variation.
introduced were not identical with the desired precision and had a radius deviation of 6 angstroms. The transmission bands and bandgaps (both in the discrete and continuous spectra) are clearly identified in Figure 9(A), while the angstrom-scale nonuniformity give rise to the appearance of localized states in the bandgap region. The uniformity of this microresonator chain was improved by the correction with shorter beam shots corresponding to a much smaller radius variation. First, the calibration of these shots was performed, when the created microresonators were successively exposed to a sequence of laser shots linearly increasing in number. The result of the calibration exposure is shown in Figure 9(B). By comparison of Figure 9(A) and (B), the radius variation introduced by a single shot equal to 0.07 angstrom was determined. Next, to make the microresonators equal, their radii were altered by exposing them to the appropriate numbers of shots. The spectral plot of a remarkably uniform microresonator chain, achieved after two iterations, is shown in Figure 9(C). The theoretical modeling of the created microresonator chain was performed using Eq. (8)–(11). The determined radius variation (solid white curve) is shown in Figure 9(D). The transmission spectrum distribution calculated with this radius variation is in excellent agreement with the experimental spectrum distribution in Figure 9(C).

Analysis of the experimental spectral distribution [Figure 9(C)] shows that, in excellent agreement with theoretical calculations, the chain of 30 coupled SNAP microresonators was fabricated with sub-angstrom precision of 0.7 angstrom and a standard deviation of 0.12 angstrom. This result surpasses the fabrication precision achieved in previously developed photonic technologies [39–45] by two orders of magnitude. An application of the SNAP coupled microresonator chain as a miniature delay line was demonstrated in [37].

In contrast to miniature slow light delay lines considered previously [39–45, 54], this delay line is not based on the periodic photonic crystal or coupled microresonator structures. Instead, it is fabricated of a bottle resonator with a nanoscale parabolic variation of its radius. The footprint of this device is only 0.12 mm² (<0.05 mm²/ns). It exhibits the record large 2.58 ns (3 bytes) dispersion-less delay of 100 ps pulses. The intrinsic (0.44 dB/ns) and full (1.12 dB/ns) loss of this device is much smaller than 10–100 dB/ns losses demonstrated for miniature delay lines to date [45].

In another example shown in Figure 11, the nanoscale fiber radius variation was introduced to imitate the word “NANO.” The height of this word is around 15 nm. This

3.4 Advanced programmed nanoscale variation of the optical fiber radius

A more general approach to introduce nanoscale variation of the effective fiber radius is performed by the programmed translation of the laser beam along the optical fiber. The beam width at the fiber surface or, more exactly, the axial width of the deformed region, determines the smallest possible axial size of the introduced features. Typically, this size is around 10 μm and is restricted by the CO₂ laser wavelength and fiber radius.

As an example, Figure 10 shows the spectral characterization of a SNAP device, which was fabricated by the translation of the focused CO₂ laser beam along the 19 μm fiber axis with a nonlinearly varied speed to arrive at the parabolic variation of the fiber radius [37]. The deviation of the introduced radius variation from the exact parabolic (white line) was <0.9 angstrom. This SNAP fiber was used in [37] to demonstrate a miniature slow light delay line with a breakthrough performance. In contrast to miniature slow light delay lines considered previously [39–45, 54], this delay line is not based on the periodic photonic crystal or coupled microresonator structures. Instead, it is fabricated of a bottle resonator with a nanoscale parabolic variation of its radius. The footprint of this device is only 0.12 mm² (<0.05 mm²/ns). It exhibits the record large 2.58 ns (3 bytes) dispersion-less delay of 100 ps pulses. The intrinsic (0.44 dB/ns) and full (1.12 dB/ns) loss of this device is much smaller than 10–100 dB/ns losses demonstrated for miniature delay lines to date [45].

In another example shown in Figure 11, the nanoscale fiber radius variation was introduced to imitate the word “NANO.” The height of this word is around 15 nm. This

![Figure 10](image1.png) The surface plot of the experimentally measured transmission amplitude of the SNAP bottle resonator with parabolic radius variation.

![Figure 11](image2.png) Imitation of the word “NANO” with nanoscale variation of the optical fiber radius.
example together with that in Figures 9 and 10 illustrate the flexibility of the developed fabrication methods.

3.5 Comparison of SNAP with previously developed photonic technologies

The advantages of the SNAP platform compared to previously developed technologies for the fabrication of miniature photonic devices (e.g., planar high and low refractive index, photonic crystal, and silica microresonators [39–45]) are outlined in Table 1. The characteristic dimensions of individual SNAP elements can be potentially as small as 10 μm, while the propagation loss and the fabrication precision of SNAP circuits is two orders of magnitude smaller than that of the previously developed photonic fabrication platforms. The success of SNAP is based on the remarkably low material loss and surface roughness of drawn silica fibers, evidence of strong localization of WGMs caused by nanoscale variations of the fiber radius, and possibility of the introduction and characterization of these variations with sub-angstrom precision.

4 Characterization of the optical microfiber and conventional fiber effective radius variation

The experimental confirmation of the nanoscale properties of optical microfibers and SNAP fibers considered above requires methods for precise characterization of effective radius variation. Characterization with required sub-nanometer and sub-angstrom precision cannot be based on conventional methods like scanning and transmission electron microscopy, atomic force microscopy, as well as scanning near-field optical microscopy and other

Table 1 Comparison of the previously developed photonic fabrication platforms with the SNAP platform.

<table>
<thead>
<tr>
<th>Platform</th>
<th>Illustration</th>
<th>Dimension of elements</th>
<th>Propagation loss</th>
<th>Fabrication accuracy</th>
<th>Integration</th>
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<td>0.1 dB/cm</td>
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<tr>
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<tr>
<td>Surface nanoscale axial photonics</td>
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<td>0.0001 dB/cm</td>
<td>0.1 nm</td>
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</tbody>
</table>
side-scattering methods due to the cylindrical geometry of optical fibers. Fortunately, simple methods for the characterization of effective radius variation of optical microfibers and regular fibers have been developed. These methods are reviewed below.

4.1 Characterization of the optical microfiber radius variation with angstrom precision

The characterization of the microfiber radius variation described in this subsection is important for many applications of microfibers and the investigation of their properties. In particular, it was employed for the determination of the taper dimensions in Subsection 2.4. In Ref. [14], a simple measurement tool for the comprehensive characterization of microfiber effective radius variation with sub-nanoscale precision was developed. The measurement setup is illustrated in Figure 12(A). It consists of a partly stripped optical fiber segment used as a probe which was positioned in direct contact with the microfiber under test normal to its axis. Generally, the fiber probe can be replaced by any light-absorbing material with a sharp edge (e.g., a metal razorblade). The microfiber studied in [14] was a waist of biconical fiber taper of 4 mm length and around 0.53 μm radius connected to the light source and detector. The probe was translated along the microfiber and, simultaneously, the transmission power was measured. For relatively small microfiber radius variation Δr_{eff}(z), the experimentally measured variation of the transmission power in the log scale, log[P(z)], is proportional to the radius variation at the point of the probe-microfiber contact:

\[ 10\Delta \log(P(z)) = C\Delta r_{eff}(z). \]  

The calibration constant was determined in Ref. [14] by comparison of the radius variation measured using scanning electron microscope (SEM) with the transmission power variation measured by scanning the probe along the microfiber. Figure 12(B) shows good correspondence between the transmission power and SEM measurements. Curves 1 and 2 in Figure 12(C) show excellent reproducibility of the effective radius variations measured along the 500 μm segment of the microfiber. The inset in Figure 12(C) shows a magnified comparison of these curves along a 100 μm segment. The RMS difference between curves 1 and 2 in Figure 12(C) is 0.37 nm, while in the inset it is only 0.19 nm. Curve 3 in Figure 12(C) characterizes the same segment rotated by 90°. It is seen that curves 1, 2 and 3 are remarkably similar. This indicates the axial symmetry.

Figure 12  (A) Illustration of the setup for characterization of the effective radius variation of a microfiber. (B) Logarithm of transmission power of the microfiber under test as a function of the probe coordinate vs. the radius variation measured by a SEM. (C) Effective radius variation along a 500 μm microfiber segment. Curves 1 and 2 (shifted along the vertical axis for visibility) demonstrate the repeatability of measurements. Curve 3 characterizes the same segment after the microfiber was rotated by 90°. Inset: magnified curves 1 and 2 along the 100 μm segment.
of the observed microdeformations. Curves 1 and 2 show two pronounced asymmetrically localized defects (sharp peaks), which are absent in curve 3, thus indicating the axial asymmetry of these defects. Using the experimental curves in Figure 12(C), it is possible to estimate the attenuation constant of the microfiber as $\alpha \approx 0.01 \text{ dB/cm}$, which was in reasonable agreement with the direct measurements $\alpha \approx 0.05 \text{ dB/cm}$ [14]. The described technique opens intriguing opportunities for the investigation of optical and physical properties of optical microfibers at nanoscale precision.

### 4.2 Characterization of conventional optical fiber radius variation with sub-angstrom precision

Characterization of the optical fiber radius along the centimeter-scale length with sub-angstrom precision has been demonstrated in [53]. Aside from general interest and application to the characterization of SNAP devices, accurate measurement of fiber radius variation is important for the improvement of fiber Bragg grating quality [55], understanding the fiber drawing process, and other applications. The previously known methods [56–58] solved this problem with an accuracy at best of a few tens nm.

The method of fiber radius characterization [53] develops the idea proposed by Birks and coauthors [52]. Figure 13(A) illustrates the experimental setup used in [53]. A biconical fiber taper with a microfiber waist was connected to a laser source and detector. The microfiber was translated relative to a tested fiber with two linear orthogonal stages. In the process of measurement, the microfiber was translated along the tested fiber and periodically touched it at contact points where the resonant transmission spectrum was measured.

Generally, the fiber radius variation $\Delta r_{\text{eff}}(z)$ can be determined from the measured transmission amplitude $S(\lambda, z)$ using the SNAP theory reviewed in Subsection 3.1 and by solving the inverse problem. In several cases, the solution of the inverse problem is simplified. For example, if the fiber radius is changing extremely slowly [53], it is proportional to the shift of the resonant peak [52]

$$\Delta r_{\text{eff}}(z) = (\lambda_{\text{res}}(z) - \lambda_{\text{res}}(0))r_{0}/\lambda_{\text{res}}.$$

Figure 13(B) shows the surface plot of the resonant transmission amplitude of the 19 μm radius optical fiber used in experiments described in Section 3. The radius of this fiber has the slope of 0.5 nm/mm and is not as uniform as the commercial fiber A considered in [53].

The smooth and elongated shape of a SNAP bottle resonator like that shown in Figure 10 can be determined in the semiclassical approximation [31]. For each microfiber position $z$, the resonant peaks of amplitude are located in the interval between the turning point wavelengths, $\lambda_{\text{turn}}(z) = \lambda_{\text{res}}(1 + \Delta r_{\text{eff}}(z)/r_{0})$ and the wavelength $\lambda_{\text{cont}}$ corresponding to the threshold between the discrete and continuous spectra, which is independent of $z$. Thus, in this approximation,

$$\Delta r_{\text{eff}}(z) = (\lambda_{\text{turn}}(z) - \lambda_{\text{res}}(0))r_{0}/\lambda_{\text{res}}.$$

To determine the fiber radius variation more accurately as well as for more general situations, the inverse problem for the Schrödinger equation, Eq. (8), can be solved numerically.

### 5 Summary and discussion

This paper reviewed situations when the transmission spectra of an optical microfiber with the radius of the order of 100 nm–1 μm and also of an optical fiber with radius of the order of 10–100 μm are sensitive to nanoscale variation of the fiber radius. For a microfiber, the considered effects are explained by strong sensitivity of the evanescent field distribution to such variations. In particular, it
was demonstrated that there exists a threshold microfiber radius, which corresponds to the transition between the waveguiding and non-waveguiding regimes. The threshold width is only several nm in microfiber radius variation.

For an optical fiber with a larger radius, the considered nanoscale effects are related to the slow propagation of WGMs along the fiber axis. These effects form the basis of SNAP, a photonic technology which enables the fabrication of super-low-loss photonic circuits at the surface of an optical fiber with unprecedented sub-angstrom precision. It has been shown that variation of just a few nanometers in the fiber radius can fully control the axial speed of WGMs and, in particular, lead to their full localization. The phenomena are simply described by the one-dimensional Schrödinger equation.

At first sight, the situations exhibiting strong sensitivity of the transmission spectra to nanoscale variation of the microfiber radius (Section 2) and to the SNAP fiber radius (Section 3) have little in common. However, they represent two limiting cases of light propagation in the optical fiber. In the case of a microfiber illustrated in Figure 14(A), the fundamental mode is primarily localized outside the microfiber, and propagation constant $\beta$ is very close to the wavenumber of the supporting optical material, $2\pi/\lambda$ (assumed here to be air with the refractive index $n$ close to unity), while the transverse propagation constant [Eq. (1)] is small, $\gamma_0 < \beta$. In contrast, the propagation constant in the case of a SNAP fiber is much smaller than the transverse propagation constant, $\beta < \beta_0$, which, in turn, is close to the wavenumber of the fiber optical material, $2\pi n_f/\lambda$. It would be interesting to find out if these two situations fully exhaust the phenomena in an optical fiber which are sensitive to the nanoscale variation of its effective radius.

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