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Research Note

Speed Gradients and the Perception of Surface Slant: Analysis is Two-dimensional not One-dimensional

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Motion parallax provides cues to the three-dimensional layout of a viewed scene and, in particular, to surface tilt and slant. For example, as a textured surface, inclined around a horizontal axis, translates horizontally relative to an observer's view point, then, in the absence of head and eye movements, the observer's retinal flow will contain a one-dimensional (1D) vertical speed gradient. The direction of this gradient indicates the direction of surface tilt, and its magnitude and sign can be used in calculating the magnitude and sign of the surface slant. Alternatively, the same retinal flow contains a 1D translating component, plus a two-dimensional (2D) component of rotation (*curl*), and a 2D component of deformation (*def*). On this view, the direction of surface tilt is related to the orientation of *def* and the magnitude and sign of the surface slant is related to the magnitude and sign of *def*. We used computer generated random dot patterns as stimuli to determine whether the human visual system employs a 1D analysis (i.e. 1D speed gradients) or a 2D analysis (i.e. deformation) of surface slant from motion parallax. Using a matching technique we found compelling impressions of slant when we vector summed a translation field with (i) vertical shear, horizontal shear or deformation (made from vertical and horizontal shear), but not rotation; and (ii) vertical compression, horizontal compression or deformation (made from vertical and horizontal compression), but much less so for expansion. In both cases, the first three conditions contain *def*, but the fourth does not, and the last three conditions contain 1D speed gradients orthogonal to the perceived axis of inclination, but the first one does not. Therefore, the results from the first and fourth conditions distinguish between the two processing strategies. They support the idea that surface slant is coded by combining both horizontal and vertical speed gradients in a way similar to the 2D differential invariant *def* and oppose the view that surface slant is encoded by a 1D analysis of motion in a direction orthogonal to the perceived axis of inclination. In a further experiment, we found essentially no effect of reducing the field size from 18 to 9 deg.

Optic flow Speed gradient Motion parallax Deformation Shear

INTRODUCTION

Motion parallax (also known as differential perspective; Koenderink & van Doorn, 1975), can provide an immediate and compelling impression of depth (Braunstein, 1968; Rogers & Graham, 1979; Braunstein & Tittle, 1988; Freeman, Harris & Meese, 1993). The relative depth cues given by motion parallax can be utilized by the visual system to determine surface tilt (the direction in which a surface is inclined) and slant (the amount by which a surface is inclined). This is illustrated in Fig. 1(A), where we consider only horizontal motion (in this case, the observer moves to the right, or equivalently, the surface moves to the left). The left-hand panel shows a surface inclined about a horizontal axis (tilt = 90 deg), and has positive slant, such that the surface rises up and away from the observer‡ (e.g. an inward opening catflap, viewed from within). The right-hand panel shows a surface inclined about a vertical axis (tilt = 0 deg), and has positive slant such that the surface becomes more distant to the right (e.g. an inward opening, right-hand saloon door, viewed from within). The surface tilted at 90 deg produces a one-dimensional (1D), vertical shearing gradient [termed horizontal shear;

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‡Our terminology is such that for a tilt of 90 deg, a surface with *negative* slant would fall away from the observer. Equivalently, such a surface could be also described as having a tilt of 270 deg and positive slant.

Fig. 1(B), left], whereas the surface tilted at 0 deg, produces a 1D, horizontal compression* gradient [termed horizontal compression; Fig. 1(B), right]. Intermediate directions of tilt produce linear combinations of horizontal compression and horizontal shear. This type of description suggests that slant could be recovered from the analysis of 1D speed gradients.

Alternatively, the same transformation can be considered in terms of two-dimensional (2D) operations [Fig. 1(C)]. Here the transformation associated with a tilt of 90 deg can be thought of as the sum of a shape change (*def*) and a rotation (*curl*)—[Fig. 1(C), left], while that associated with a tilt of 0 deg is the sum of a shape change (*def*) and an expansion (*div*)—[Fig. 1(C), right]. Note that the *def* component in the surface tilted at 0 deg, is rotated through 45 deg relative to that in the surface tilted at 90 deg. Although less immediately obvious, this type of description, in terms of the 2D differential invariants *div*, *def* and *curl*, can be used to provide a complete description of general optic flow (Koenderink, 1986; Koenderink & van Doorn, 1991), which captures the effects of both 3D surface layout and of observer locomotion. In particular, for example, the amplitude and direction of deformation (*def*), conveniently encodes surface slant and tilt, respectively, irrespective of the effects of eye movements (Koenderink, 1986).

A fact that we shall make use of in our methods is that the 2D operations, *div*, *def* and *curl*, can all be thought of as combinations of 1D shearing and compression gradients (see the Appendix). First recall from Fig. 1 that horizontal shear [Fig. 2(A)] is made from *def* [Fig. 2(C)] plus *curl* [Fig. 2(D)], while vertical shear [Fig. 2(B)] is made from *def* minus *curl*. Viewed this way, it is easy to see that when 1D horizontal and 1D vertical shear [Fig. 2(A,B)] are summed, the curl components cancel, leaving only the *def* component, and so produce diagonal deformation [Fig. 2(C)]. Similarly, when vertical shear is subtracted from horizontal shear, the *def* components cancel, leaving only *curl*, and so produce rotation [Fig. 2(D)]. Likewise, Fig. 8 shows how the sum of a 1D horizontal and a 1D vertical compression produces 2D expansion [*div*, Fig. 8(D)], while their difference produces 2D vertical deformation [Fig. 8(C)].

In this paper we confine our interest to the recovery of surface slant and concentrate first on the horizontal motion of a surface inclined about a horizontal axis [e.g. the surface depicted in Fig. 1(A)]. The resulting motion parallax can be equally well described in terms of a simple 1D shearing gradient [Fig. 1(B)] or as the combination of 2D operators [Fig. 1(C)]. Mathematical accounts (e.g. Koenderink, 1986), tend to make use of the 2D description, whereas most empirical studies of motion parallax and perceived slant have emphasized the 1D gradients (e.g. Rogers & Graham, 1979; Rogers & Collett, 1989; Braunstein & Tittle, 1988; Braunstein, Liter & Tittle, 1993; though see Freeman, Harris & Meese, 1993). Which of these descriptions better captures the analysis performed in the human visual system? Here we provide evidence that the 2D description is more appropriate.

*We use the term compression, to refer to 1D compression/expansion, in order to avoid confusion with 2D contraction/expansion, which we refer to as expansion (*div*). Our sign convention is such that positive compression is a 1D expanding pattern.

1D and 2D slant analyses do not always give the same results

The above observations on 1D and 2D flow patterns lead to specific predictions of the perceived slant of optic flow patterns. Moreover, these predictions can distinguish between the use of a 1D or a 2D analysis by the visual system. For example, consider the stimuli shown in Fig. 2 (the equations describing the speed gradients of these stimuli are in the Appendix),

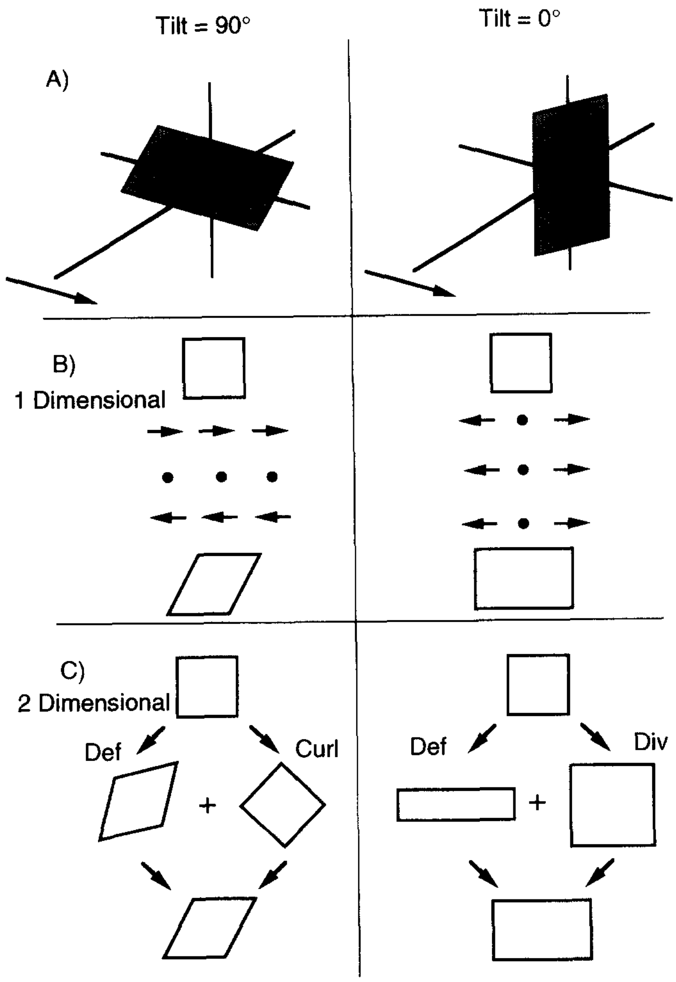

FIGURE 1. Surface slant can be encoded by a 1D analysis of speed gradients, or a 2D analysis of deformation, for surfaces inclined about a horizontal axis (tilt = 90 deg, left panels) and a vertical axis (tilt = 0 deg, right panels). (A) Pictorial representations of two textured and slanted surfaces, each viewed by an observer moving rightwards. (B) For tilt = 90 deg, the optic flow will contain a vertical speed gradient, termed horizontal shear. For tilt = 0 deg, the optic flow will contain a horizontal speed gradient termed horizontal compression. (C) Although the patterns of optic flow illustrated in panel (B) are 1D, they can each be thought of as built from a pair of 2D transforms. For tilt = 90 deg these transforms are *def* and *curl*, while for tilt = 0 deg, the transforms are *def* and *div*. Note that the *def* component for a tilt of 0 deg is rotated through 45 deg relative to the *def* component for a tilt of 90 deg.

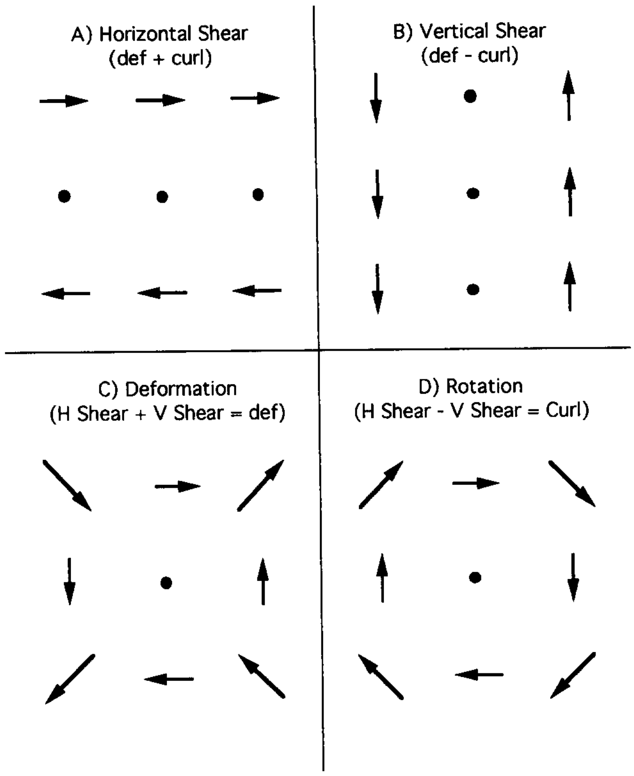

FIGURE 2. The combination of shearing components used in Expts 1 and 2. (A) Horizontal shear is 1D, though it is made from a pair of 2D components, *def* + *curl*. (B) Vertical shear is also 1D, and is made from a different combination of the same pair of 2D components found in horizontal shear, (*def* − *curl*). (C) A 2D deforming pattern contains only *def*, and can be made by summing horizontal shear and vertical shear. In this case, the two *curl* components are of opposite sign and so cancel, leaving the two positive *def* components. (D) A 2D rotating pattern contains only *curl*, and can be made by subtracting vertical shear from horizontal shear. In this case, the two *def* components have opposite sign and so cancel, leaving the two positive *curl* components.

and slant about a horizontal axis. The 1D account of perceived surface slant utilizes the vertical speed gradient, and so predicts that the flow patterns shown in Fig. 2(A) (horizontal shear), Fig. 2(C) (deformation) and Fig. 2(D) (rotation) should all appear slanted, because all of these patterns contain a vertical speed gradient (i.e. a component of horizontal shear). However, vertical shear [Fig. 2(B)], does not contain a vertical speed gradient, and so should not appear slanted. On the other hand, a 2D account of perceived slant makes quite different predictions. Under this hypothesis, horizontal shear, vertical shear, and deformation, should all appear slanted because they all contain a component of *def*, while rotation contains no such component, and so should not appear slanted. Thus, the two crucial conditions that distinguish between the two hypotheses (1D analysis of speed gradients, or 2D analysis of deformation) are those of rotation and vertical shear.

In a recent paper on stereopsis, Howard and Kaneko (1994) utilized the above predictions by using dichoptic texture displays with either horizontal disparity alone, vertical disparity alone, horizontal disparity plus vertical disparity (deformation; see Fig. 2), or horizontal disparity minus vertical disparity (rotation; see Fig. 2). They concluded that the perceived inclination of a large isolated textured surface is derived from the combination of both horizontal and vertical shear disparities (deformation). The basic geometry underlying stereopsis and motion parallax is of course the same. Thus, while Howard and Kaneko (1994) presented dichoptic displays and manipulated vertical and horizontal disparities, we presented monocular displays and manipulated vertical and horizontal speed gradients. Furthermore, we extended our experiments beyond those performed by Howard and Kaneko, by also manipulating horizontal and vertical compression gradients (see Figs 8 and 9).

In a similar way to Howard and Kaneko (1994), we are able to reject the hypothesis that the vertical (or horizontal) speed gradient alone is used to determine the perceived slant of a surface inclined around a horizontal (or vertical) axis, and conclude that the visual system employs a measure that utilizes both vertical and horizontal speed gradients in a similar way to the differential invariant *def* (deformation).

Surface synthesis from translation and differential invariants

In all of our experiments, we included a component of horizontal translation to simulate the lateral translation of a textured surface moving relative to an observer (see Fig. 1). Moreover, the inclusion of this translating component removed ambiguity over the sign of the perceived slant. For example, horizontal shear alone

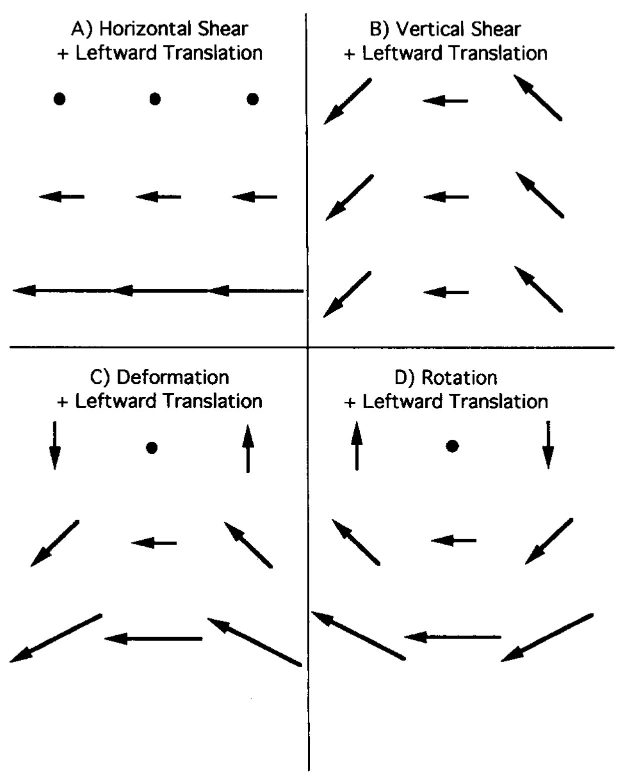

FIGURE 3. The four optic flow conditions used in Expts 1 and 2. These are the same components as those shown in Fig. 2, but with the addition of a leftward (negative) translating component. A vertical speed gradient exists in horizontal shear (A), deformation (C) and rotation (D), but is absent in vertical shear (B). Observers matched the perceived slant of these patterns to the surface of a pictorial wire-frame cube that could be rotated about a horizontal axis.

[Fig. 2(A)] can appear as a slanted planar surface (e.g. an open catflap) that yaws back and forth around a horizontal axis. However, this stimulus provides no cue to whether the surface falls down, away from the viewpoint (e.g. a catflap pushed outward when viewed from within), or rises up, away from the viewpoint (e.g. a catflap pushed inward when viewed from within). Our own observations in the laboratory have shown that the visual system is capable of both these percepts, and that with a bit of effort, the observer can switch between the two. In this respect, the stimulus gives rise to a bistable percept, similar to that of the Necker Cube.* The vector addition of a horizontal translating field removes this ambiguity because the speeds of the dots at the top and the bottom of the stimulus are no longer the same—the edge (top or bottom) with the greater speed is typically perceived as nearer. Figure 3 illustrates the effect of adding leftward (negative) translation† to the four components shown in Fig. 2. Note that the addition of this component does not change the presence or absence of speed gradients and differential invariants within the stimuli.

The horizontal shear and translation schematized in Fig. 3(A) simulates the motion parallax in a slanted planar surface, tilted at 90 deg and translating laterally relative to the observer (Fig. 1, left); i.e., a polar projection of a real slanted planar surface onto a perspective plane (e.g. the display screen).‡ By manipulating the number and type of differential invariants that are contained within the stimulus, the 'original' surface [Fig. 3(A)] is perceptually changed. For example, observations of stimuli like those in Fig. 3 have revealed that (i) some surfaces appear planar, whereas others have a curved horizontal cross-section, and (ii) some surfaces yaw and roll as well as translate. Nevertheless, all four of the flow patterns used in Expts 1 and 2 (shearing components), and schematized in Fig. 3, appeared as essentially rigid surfaces with a tilt of 90 deg.

In Expt 3, a slanted planar surface tilted at 0 deg, and translating laterally relative to the observer (Fig. 1, right) was simulated by adding horizontal translation to horizontal compression [Fig. 3(A)]. Here, even this 'original' surface texture tended to appear as non-rigid when the magnitude of the compression was high—we also found this to be so for the stimuli schematized in Fig. 9(B,C). Sometimes, this non-rigidity appeared as a slight bending of the surface (cf. bending deformation, Koenderink, 1986), but often, the surface appeared as a pitching and/or looming and translating planar surface, with texture elements that slid around upon this surface.

A more detailed analysis of the perceptual nuances of our flow patterns is clearly an empirical issue, and one which is currently receiving our attention. However, here we concern ourselves only with the perception of slant.

EXPERIMENT 1: PERCEIVED SLANT ABOUT A HORIZONTAL AXIS

Methods

Stimuli

The stimuli were generated by a PC-type computer and displayed via a CED 1401-plus laboratory interface upon the screen of an oscilloscope (Hewlett Packard 1304, P31 phosphor) at a frame rate of 50 Hz. The stimuli consisted of a set of 599 randomly-positioned dim dots, each of constant size. These dots moved back and forth (see below) behind a fixed circular window with a diameter subtending 18 deg from a viewing distance of 57 cm. The display surround was black.

The motion of the dots was determined by the vector summation of either two or three vector component fields depending upon the condition (see below). One of these components was always a horizontal translating field while the other two components were a horizontal shear field and a vertical shear field [Fig. 2(A,B)]. All three of the vector fields were modulated sinusoidally at 1 Hz. When presented alone, the translating component produced 1 Hz horizontal sinusoidal motion, with each dot travelling a peak-to-peak distance of 1.42 deg.

The shear fields contained a 1D velocity gradient in either the vertical direction [horizontal shear; Figs 2(A), 3(A)] or the horizontal direction [vertical shear; Figs 2(B), 3(B)]. More formally, we define screen coordinates (x, y) to have the origin in the centre of the circular window and define the instantaneous position of each individual dot as $(x + sgy, y)$ in a horizontal shear field and as $(x, y + sgx)$ in a vertical shear field, where x and y are the randomly chosen horizontal and vertical screen coordinates for the starting position of each dot, g is the magnitude of the speed gradient $(0 \leqslant g \leqslant 1)$ and s is the 1 Hz sinusoidal modulation ($s = \sin[2\pi t]$; where t is time in seconds). Thus, the direction of motion for each dot in the shear field was rightwards (for positive sgy) and leftwards (for negative sgy) for horizontal shear, and upwards (for positive sgx) and downwards (for negative sgx) for vertical shear. Further, the speed of motion for each dot in the field was proportional to gy for horizontal shear and gx for vertical shear.

We used four different conditions of shear as follows: (a) horizontal shear, (b) vertical shear, (c) horizontal shear plus vertical shear (deformation), and (d) horizontal shear minus vertical shear (rotation). These four different types of shear field are shown in Fig. 2, and correspond to the four different types of shear disparity

*The purely deforming flow-pattern shown in Fig. 2(C) is even more ambiguous: this stimulus can appear as a rigid slanted surface that yaws and rolls at one of four different angles of tilt (0, 90, 180, 270 deg), or as a slightly less rigid surface that pitches at four yet different angles of tilt (45, 135, 225, 315 deg).

†In order to facilitate comparison between our own data and those of Howard and Kaneko, we chose to add *negative* translation to our displays in order that *positive* shear (see Methods) should produce *positive* perceived slant.

‡For clarity of presentation, the velocity gradients shown in Figs 2, 3, 8 and 9 are much larger than those actually used in the experiments. One consequence of this is that Fig. 3(A), for example, contains a horizontal array of stationary points. Such a situation is not realisable with a purely translating, physical stimulus plane and a fixed observer. However, the magnitudes of both the translation component and the differential invariants that we used ensured that none of our stimuli contained stationary points.

used by Howard and Kaneko (1994; their Fig. 2). In comparing our own experiments with those of Howard and Kaneko (1994), it is important to note that those authors used a different sign convention from us (the sign of their vertical shear is the reverse of ours), with the result that in their study, deformation is the difference between the two shear components, and curl is the sum of the two shear components. We stress that this difference is one only of convention and does not reflect a difference in experimental conditions or results.

Procedure and response measures

Two of the authors (TSM and MGH) and a naive observer (JH) undertook the experiment. Viewing took place in a darkened room, though the frames of both the stimulus and graphics displays were dimly visible. The observer's head was supported by a chin and head rest which also contained a blanking-piece that could be swung down in front of the observer's non-preferred eye. With the blanking piece in position, all stereoscopic cues to flatness were removed from the display. On each trial the subject was presented with a moving dot stimulus from a pseudo-randomly selected condition and with a pseudo-randomly selected level of speed gradient in the range 0–0.05 (0–5%).

The subject's task was to match the perceived slant of the stimulus with the front surface of a polar projected wire-framed cube that could be rotated on the PC screen alongside the main display, using two mouse buttons. This response stimulus is naturally interpreted as an object in a specific pose and thus encourages a 3D match. We find this response procedure less cumbersome than the traditional manipulation of a real planar surface (e.g. Braunstein, 1968; Howard & Kaneko, 1994), though in control experiments we have found that the two methods give indistinguishable results. While making the adjustment, the subject was free to view the stimulus for as long as he wished. The match was recorded and the next trial initiated by pressing a third response button.

Each session consisted of 32 trials (eight trials at evenly spaced levels of shear from each of the four conditions) and each subject performed a total of three sessions. The levels of shear were interleaved between sessions to give a total of 24 different levels of shear (12 positive and 12 negative) for each condition.

Results and Discussion

Matched slant is shown as a function of 1D speed gradient (one component of shear) in Fig. 4 for TSM (left panels) and MGH (right panels), and in Fig. 5 for the naive observer (JH). The top panels show results for conditions of horizontal shear (■) and vertical shear (△), and the bottom panels show results for conditions of deformation (◆) and rotation (○). Although the magnitude of matched slant is typically greater for JH, the overall trend of results is similar for all three observers.

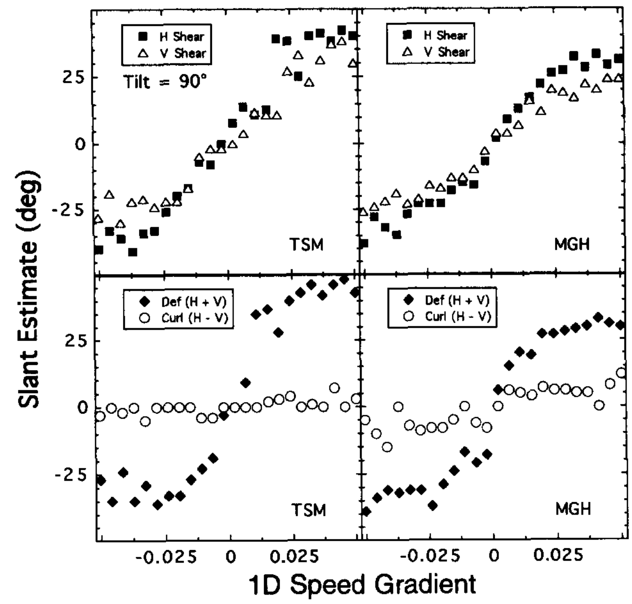

FIGURE 4. Perceived slant as a function of 1D speed gradient (i.e. the value of a single component of shear) for TSM (left panels) and MGH (right panels). The slanted surfaces were perceived to be inclined about a horizontal axis. The top panels show the results for the pair of 1D conditions (horizontal shear, ■; vertical shear, △) and the bottom panels show results for the pair of 2D conditions (deformation [*def*], ◆; rotation [*curl*], ○). In all panels, the conditions denoted by filled symbols would be expected to produce a perception of slant, regardless of whether the visual system uses a 1D analysis of speed gradients, or a 2D analysis of deformation. However, the conditions denoted by the open symbols are those that can distinguish between the two different possibilities. A 1D analysis predicts that there should be no effect for vertical shear (△), but that rotation (○) should appear slanted, while a 2D analysis makes the opposite prediction. The data support the hypothesis that slant perception is determined by a 2D analysis.

First we note that the only condition that failed to produce a consistent perception of surface slant was the rotation condition. All three of the other conditions (horizontal shear, vertical shear and deformation), produced functions of essentially sigmoidal

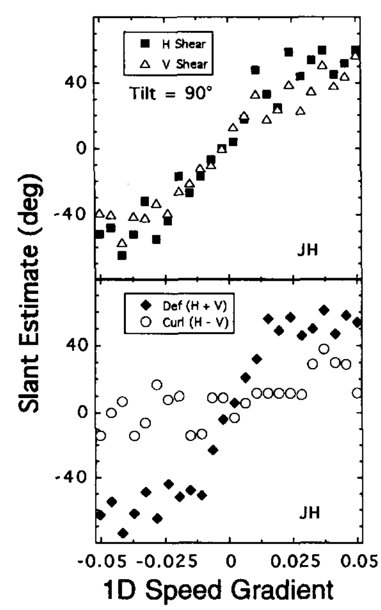

FIGURE 5. The same as Fig. 4, but for a naive observer (JH).

form. Indeed, we have modelled the perceived slant of deforming random dot patterns with a function based on *arctan* elsewhere (Freeman *et al.*, in press; also see Braunstein *et al.*, 1993).

Both the 1D and 2D hypotheses predict that horizontal shear and deformation, should both appear slanted and is indeed what we have found. However, the findings that vertical shear appears slanted, and this rotation does not appear slanted, independently support the 2D hypothesis and stand counter to the 1D hypothesis. What is more, these results are the same as those found by Howard and Kaneko (1994) in their experiment on stereopsis, and serve to illustrate further the similarities between stereopsis and motion parallax (Rogers & Graham, 1982, 1983, 1985).

EXPERIMENT 2: EFFECTS OF AREA AND DOT DENSITY

In further experiments, Howard and Kaneko (1994) went on to investigate the effects of field size on perceived slant in stereopsis, and found results similar to ours, for field diameters of 60 and 30 deg, but not for a field size of 10 deg. Rather, when the field size was small, the stimuli with rotation disparity appeared slanted (though not as much as in the other conditions), while those with vertical shear disparity appeared to have only marginal slant. In other words, Howard and Kaneko's (1994) results suggest that in stereopsis, a 2D analysis (i.e. deformation) is used for large displays, but an effectively 1D analysis (i.e. horizontal disparity alone) is used for small displays. In Expt 1, we used a circular window with a diameter of 18 deg (the largest that we could reasonably use with our equipment and a viewing distance of 57 cm). In Expt 2, we reduced the window size to a diameter of 9 deg (smaller than Howard & Kaneko's smallest condition), to see if the perception of slant determined by motion parallax is dependent upon field size as it is for stereopsis.

Methods

In most respects, Expt 2 was the same as Expt 1, except for the following changes. Two subjects (TSM and MGH) performed only two sessions each, giving a total of 16 trials for each condition. The field size was reduced from a diameter of 18 to 9 deg, while keeping the dot density the same as it was in Expt 1. However, this reduced the total number of dots in the display, so TSM performed two interleaved conditions of the experiment. In one condition, the density of dots was the same as that used in Expt 1, while in the other condition, the *number* of dots was the same as in Expt 1.

Results and Discussion

The results are shown in Fig. 6 for TSM (left panels) and MGH (right panels) for a field diameter of 9 deg with the same dot density as in Expt 1. The layout and symbols of Fig. 6 are the same as those in Fig. 4. Unlike the results of Howard and Kaneko (1994), these data are not dissimilar to those found in a large field condition (Expt 1, Fig. 4), though TSM's data for the rotation condition (○, bottom left panel of Fig. 6), now show a gentle slope, whereas they did not before (○, bottom left panel of Fig. 4). However, Fig. 7 shows the results for the same subject, for a field size of 9 deg but with *dot number*, matched to that in Expt 1. The results for the two shear conditions (top panel) and deformation (◆, bottom panel) are similar to before (Fig. 6; left panels), but now the shallow slope in the rotation condition (○) is reduced considerably.

The results of this experiment suggest that, unlike stereopsis, field size does not have a dramatic effect on the perception of surface slant from motion parallax cues, at least, not over the two field sizes (18 and 9 deg) that we tested.

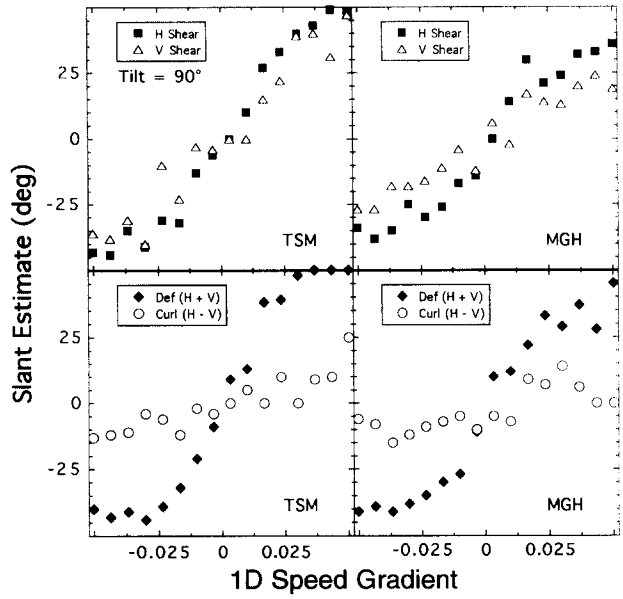

FIGURE 6. The same as Fig. 4, only the field size was reduced from 18 to 9 deg. The dot density was the same as in Expt 1.

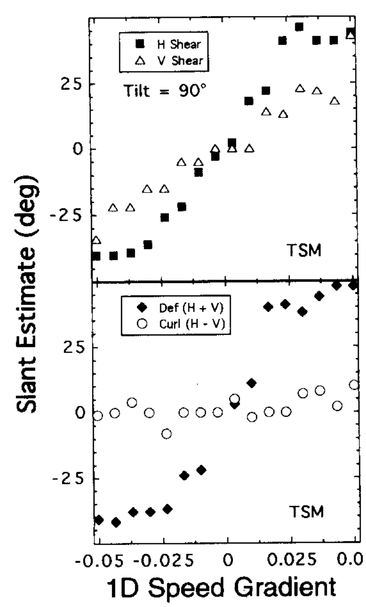

FIGURE 7. The same as the left-hand panels of Fig. 6, but instead of matching the dot density to Expt 1, the number of dots was matched.

EXPERIMENT 3: PERCEIVED SLANT ABOUT A VERTICAL AXIS

Figure 1 (left) shows that a surface inclined about a *horizontal* axis can be thought of as containing a 1D vertical speed gradient (horizontal shear) or a 2D *def* component, oriented such that the horizontal *shear* in the *def* component is aligned with the speed gradient in the surface. On the other hand, Fig. 1 (right) shows that a surface slanted about a *vertical* axis, also contains a 1D speed gradient, but that in this case, the gradient is horizontal and *compressive*. An alternative view is that this surface contains a 2D *def* component, oriented such that the horizontal *compression* in the *def* component is aligned with the speed gradient in the surface. The results of Expts 1 and 2 showed that the perceived slant of a surface inclined about a horizontal axis was related to a combination of both horizontal and vertical *shear* (deformation). This led us to ask whether the perceived slant of a surface inclined around a vertical axis is dependent upon both horizontal and vertical *compression* (2D analysis; deformation), or horizontal speed gradients alone (1D analysis; horizontal compression). The logic of the experiment is identical to that of Expt 1. In this case, the two 1D stimuli were horizontal compression alone [Fig. 8(A)] and vertical compression alone [Fig. 8(B)], and the two combinations of these 1D stimuli give deformation [Fig. 8(C)] and expansion [Fig. 8 (D)]. The addition of leftward translation to these components is shown in Fig. 9. If perceived slant is determined by a 1D analysis, then flow fields containing horizontal compression, deformation, and expansion should all appear slanted, whereas vertical compression should not. Alternatively, if perceived slant is determined by a 2D analysis, then conditions of horizontal compression, vertical compression and deformation should appear slanted, whereas expansion should not. Thus, the conditions which can distinguish between these two possibilities are those of vertical compression and expansion.

Methods

This experiment was the same as Expt 1, except for the orientation of the pictorial wire-frame cube (now rotatable around a vertical axis), and the construction of the two, non-translating flow fields (see the Appendix for the equations). Here, compression fields were used instead of shear fields. The compression fields contained a 1D velocity gradient in either the horizontal direction [horizontal compression; Fig. 8(A)] or the vertical direction [vertical compression; Fig. 8(B)]. The instantaneous position of each individual dot is given by $(x + sgx, y)$ in a horizontal compression field and by $(x, y + sgy)$ in a vertical compression field, where the variables are as defined in Expt 1 (Methods). The four conditions used in this experiment were: (a) horizontal compression, (b) vertical compression, (c) horizontal compression minus vertical compression (deformation), and (d) horizontal compression plus vertical compression (expansion)—see Figs 8 and 9.

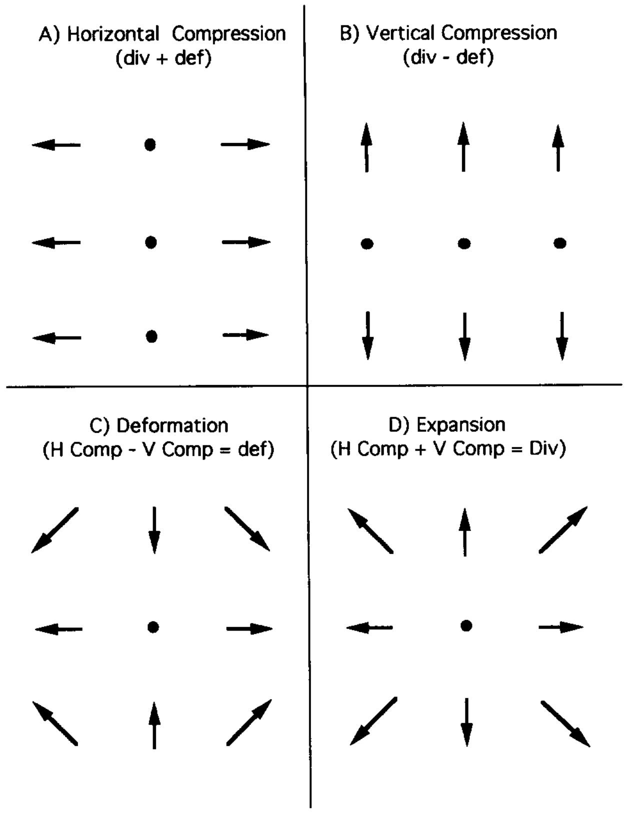

FIGURE 8. The combination of compressive components used in Expt 3. (A) Horizontal compression is 1D, though it is made from a pair of 2D components, *div* + *def*. (B) Vertical compression is also 1D, and is made from a different combination of the same pair of 2D components found in horizontal compression, (*div* − *def*). (C) A 2D deforming pattern contains only *def*, and can be made by subtracting vertical compression from horizontal compression. In this case, the two *div* components are of opposite sign and so cancel, leaving the two positive *def* components. (D) A 2D expanding pattern contains only *div*, and can be made by summing vertical compression with horizontal compression. In this case, the two *def* components have opposite sign and so cancel, leaving the two positive *div* components.

Some of the surfaces perceived in this experiment appeared to pitch slightly, as well as to loom and translate. Consequently, the perceived slant tended to vary sinusoidally with the pitch. However, observers found that they could respond consistently to these stimuli, by basing the match on the time averaged perception of slant, for example.

Results and Discussion

Figure 10 shows the results for TSM (left panels) and MGH (right panels), for horizontal and vertical compression (top panels; ■ and ◇, respectively) and deformation and expansion (bottom panels; ◆ and ○, respectively). Figure 11 shows similar results for the naive observer (JH), though once again, the magnitude of matched slant is typically greater for this observer. In a similar way to Expt 1, the data from the vertical compression, horizontal compression and deformation conditions are essentially sigmoidal in form, and reflect the compelling nature of the perceived slant in those conditions. Note that for vertical compression, the slope of the data is the reverse of that in the other two conditions, because in this condition the deformation was *negative* [see Fig. 8(B)]. The finding that vertical compression alone (with the addition of a translating component), leads to a perception of surface slant, supports the idea that slant is determined by a 2D analysis and not a 1D analysis. The results from the expansion condition (*div*) are rather less clear. For TSM, this condition produces a surface that appears flat (Fig. 10, bottom left), while for MGH and JH, the data have a shallow positive slope, though in both cases, they are rather noisy. Thus, TSM's data from the *div* condition support the 2D hypothesis, while the data of MGH and JH, are perhaps suggestive of some 1D processing. However, for what they are worth, casual observations (by TSM and MGH) confirmed that any impression of slant that could be extracted from the *div* condition, was typically not compelling or immediate, but rather, somewhat inferred. Furthermore, other studies that have employed a stimulus made from the vector sum of an expanding pattern and a translating pattern have not reported that this stimulus appears slanted (Duffy & Wurtz, 1993; Meese, Smith & Harris, 1995), though it should be borne in mind that both of these studies used binocular viewing.

Overall, the results from this experiment support the 2D hypothesis. This was so for four out of six of the data sets that addressed the issue (all three observers for vertical compression, and TSM for *div*), while the fifth and sixth data sets (MGH, *div* & JH, *div*), were less conclusive.

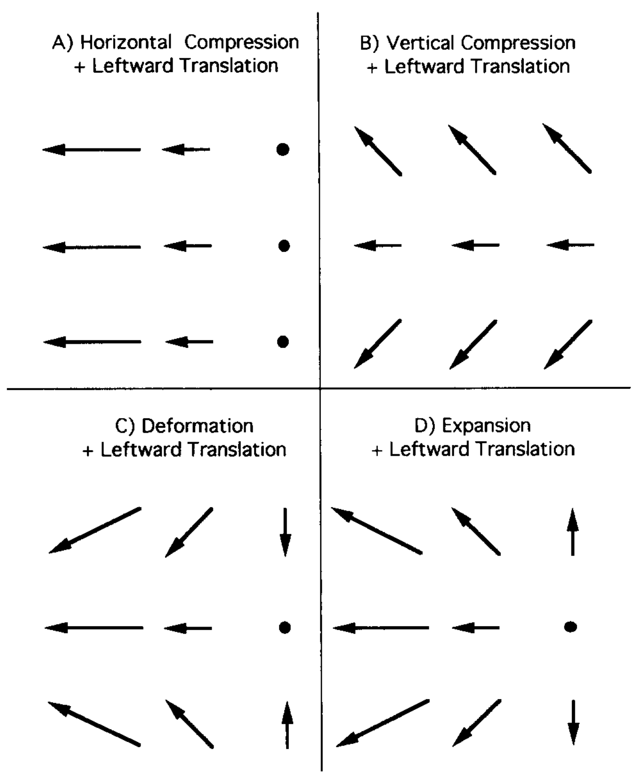

FIGURE 9. The four optic flow conditions used in Expt 3. These are the same components as those shown in Fig. 8, but with the addition of a leftward (negative) translating component. A horizontal speed gradient exists in horizontal compression (A), deformation (C) and expansion (D), but is absent in vertical compression (D). Observers matched the perceived slant of these patterns to the surface of a pictorial wire-frame cube that could be rotated about a vertical axis.

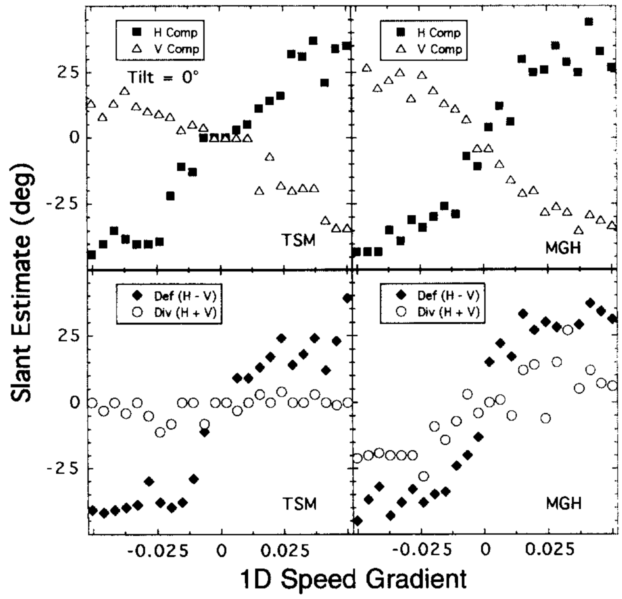

FIGURE 10. Perceived slant as a function of 1D speed gradient (i.e. the value of a single component of compression) for TSM (left panels) and MGH (right panels). The slanted surfaces were perceived to be inclined about a vertical axis. The top panels show the results for the pair of 1D conditions (horizontal compression, ■; vertical compression, △) and the bottom panels show results for the pair of 2D conditions (deformation [*def*], ◆; expansion [*div*], ○). In all panels, the conditions denoted by filled symbols would be expected to produce a perception of slant, regardless of whether the visual system uses a 1D analysis of speed gradients, or a 2D analysis of deformation. However, the conditions denoted by the open symbols are those that can distinguish between the two different possibilities. A 1D analysis predicts that there should be no effect for vertical compression (△), but that expansion (○) should appear slanted. A 2D analysis predicts that there should be no effect for expansion (○), while vertical compression (△) should appear slanted, but with an opposite sign to the other conditions. The data support the hypothesis that slant perception is determined by a 2D analysis.

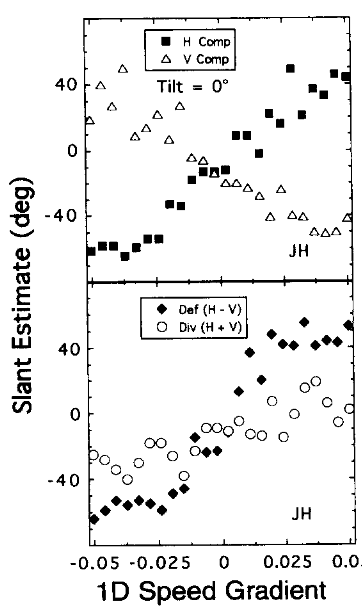

FIGURE 11. The same as Fig. 10, but for a naive observer (JH).

GENERAL DISCUSSION

Our findings clearly reject the idea that perceived slant is determined by a 1D analysis, and support the hypothesis that the perception of surface slant is determined by a combination of vertical and horizontal speed gradients. All of our results from Expts 1 and 2 suggest that this is so for a surface inclined about a horizontal axis, and most of our results from Expt 3 suggest that this is also so for a surface inclined about a vertical axis.

However, it remains unclear whether 2D information is extracted directly, by mechanisms sensitive to *def*, or indirectly, by making comparisons between pairs of 1D mechanisms sensitive to orthogonal orientations of shear and compression for example. It is tempting to posit the former, because a pair of *def* mechanisms (one oriented at 45 deg relative to the other), could be usefully employed in the analysis of surface slant for all axes of surface inclination (i.e. all tilts). However, although our data are qualitatively compatible with this scheme, they do not provide quantitative support for the idea. For example, despite containing common magnitudes of *def* component, a given level of horizontal shear (or compression), did not produce the same magnitude of perceived slant as a corresponding level of vertical shear (or compression)—compare the △ (vertical shear) with the ■ (horizontal shear), in Figs 4, 5, 6 and 7. It is noteworthy that this is also true of a comparison of the data from Howard and Kaneko's conditions of vertical and horizontal disparity (Howard & Kaneko, 1994). On the other hand, we cannot rule out the possibility that the *def* component is indeed extracted by a mechanism tuned to deformation, and that the differences in perceived slant for horizontal and vertical shear are due to later processing. Indeed, it remains to be seen how the perceived magnitude of surface slant is affected by other stimulus attributes such as yaw, pitch and roll. For example, Braunstein *et al.* (1993) have recently found that slant is persistently underestimated for polar projected linear surface trajectories (e.g. our horizontal shear condition; also see Freeman *et al.*, in press), whereas it is overestimated for orthographic projections of circumferential trajectories, which cause roll (tilt = 90 deg) or pitch (tilt = 0 deg).

We are considering these issues in our current work.

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APPENDIX

At the request of an anonymous referee, we present the equations describing the velocity gradients in our optic flow stimuli. Let x and y be the spatial coordinates of the optic flow field, and let u and v be the horizontal and vertical components of the flow field velocity vectors. Horizontal and vertical velocity gradients are given by first order partial derivatives of u and v with respect to x and y respectively. In some cases, either u or v is zero, and so partial derivatives of u and v are also zero. For all equations, k is constant.

Tilt = 90 deg

Horizontal shear [Fig. 2(A)]

$$\partial u/\partial x = 0 \quad \text{(A1a)}$$

$$\partial u/\partial y = k \quad \text{(A1b)}$$

$$v = 0 \quad \text{(A1c)}$$

Vertical shear [Fig. 2(B)]

$$u = 0 \quad \text{(A2a)}$$

$$\partial v/\partial x = k \quad \text{(A2b)}$$

$$\partial v/\partial y = 0 \quad \text{(A2c)}$$

Deformation [Fig. 2(C)]

$$\partial u/\partial x = 0 \quad \text{(A3a)}$$

$$\partial u/\partial y = k \quad \text{(A3b)}$$

$$\partial v/\partial x = k \quad \text{(A3c)}$$

$$\partial v/\partial y = 0 \quad \text{(A3d)}$$

Rotation [Fig. 2(D)]

$$\partial u/\partial x = 0 \quad \text{(A4a)}$$

$$\partial u/\partial y = -k \quad \text{(A4b)}$$

$$\partial v/\partial x = k \quad \text{(A4c)}$$

$$\partial v/\partial y = 0 \quad \text{(A4d)}$$

Tilt = 0 deg

Horizontal compression [Fig. 8(A)]

$$\partial u/\partial x = k \tag{A5a}$$

$$\partial u/\partial y = 0 \tag{A5b}$$

$$v = 0 \tag{A5c}$$

Vertical compression [Fig. 8(B)]

$$u = 0 \tag{A6a}$$

$$\partial v/\partial x = 0 \tag{A6b}$$

$$\partial v/\partial y = k \tag{A6c}$$

Deformation [Fig. 8(C)]

$$\partial u/\partial x = -k \tag{A7a}$$

$$\partial u/\partial y = 0 \tag{A7b}$$

$$\partial v/\partial x = 0 \tag{A7c}$$

$$\partial v/\partial y = k \tag{A7d}$$

Expansion [Fig. 8(D)]

$$\partial u/\partial x = k \tag{A8a}$$

$$\partial u/\partial y = 0 \tag{A8b}$$

$$\partial v/\partial x = 0 \tag{A8c}$$

$$\partial v/\partial y = k \tag{A8d}$$